* start w/ speeding question*

Jan. 24, 2014

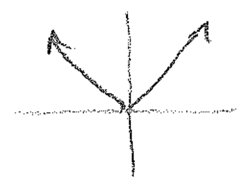
Recall: f continuous at a: $\lim_{x\to a} f(x) = f(a)$

f diff'ble at a: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists

FACT: If f is diff'ble at a, the f is cont at a.

* So if f is not cont. at a, it's not diff'ble at a.

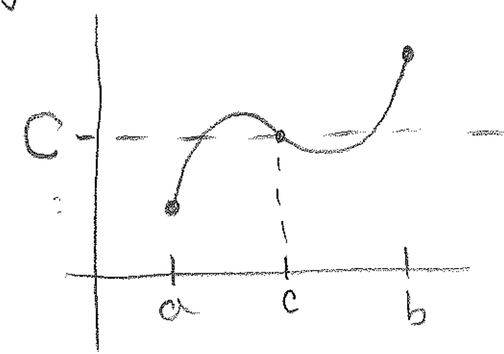
continuous but not diff'ble:



RECALL: Intermediate Value THM:

- f cont on [a,b]
- C is a real # w/ $f(a) < C < f(b)$ OR $f(b) < C < f(a)$

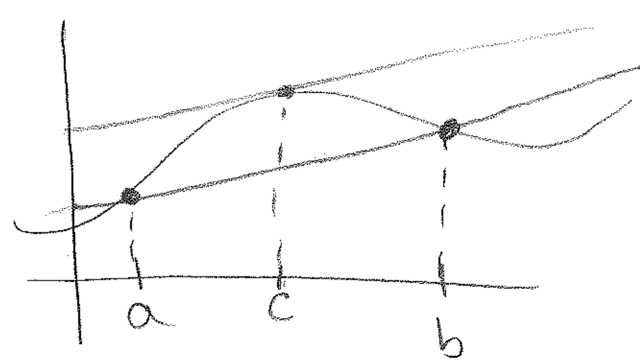
IVT says: $\exists$ c in [a,b] with $f(c) = C$



The Mean Value Theorem

Suppose f is defined and continuous on closed int. [a,b] and f' exists on open int (a,b).
Then there exists a pt. c in (a,b) ($a<c<b$) such that

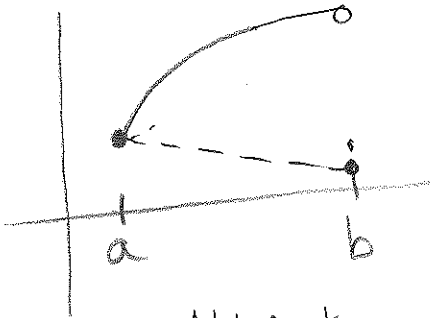
$$\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{Secant line through } (a,f(a)) \text{ and } (b,f(b))} = f'(c)$$



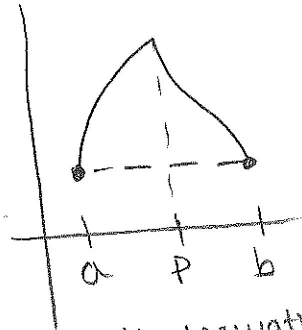
A Scenario: Driving between toll booths,
tickets are time stamped.

If avg vel is 75 mph... at some pt, you must have been going 75 mph.

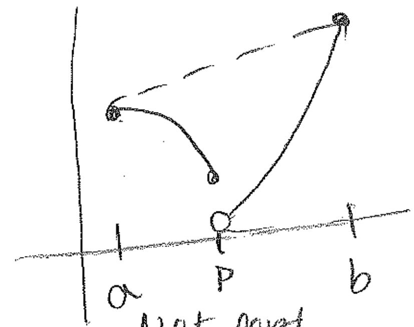
Requirements: cont on $[a,b]$
diff'ble on (a,b)



Not cont at b.



No derivative at p



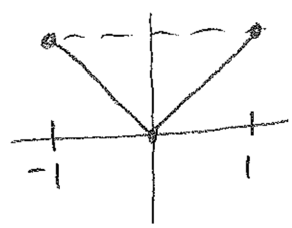
Not cont at p

Ex: Does MVT apply to $f(x)=|x|$ on $[-1,1]$?

No, not diff'ble at 0

$$\frac{|1|-|-1|}{1-(-1)} = \underline{0}$$

slope is never 0.

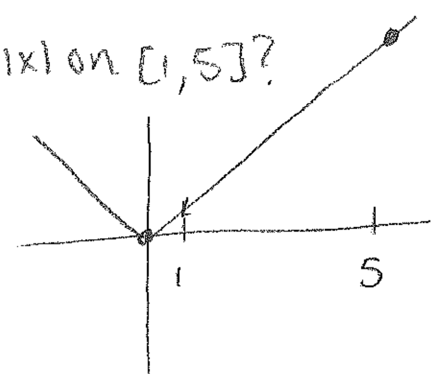


Ex: does MVT apply to $f(x)=|x|$ on $[1,5]$?

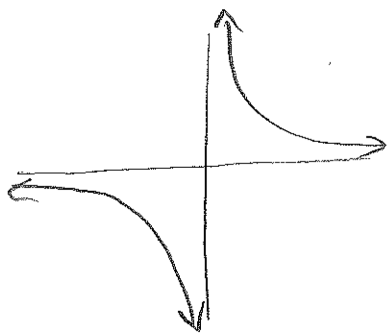
Yes

$$\frac{|5|-|1|}{5-1} = \underbrace{1}$$

slope of tangent line will be 1 at some pt.



Ex: For what intervals does the MVT apply to $f(x)=\frac{1}{x}$?



Need to avoid 0.

Any interval that doesn't include 0.

Ex: Verify MVT for $f(x)=(x+1)^3-1$ on interval $[-3,1]$

- cont on $[-3,1]$ ✓
- diff'ble on $(-3,1)$ ✓

avg rate of change $= \dfrac{((1+1)^3-1)-((-3+1)^3-1)}{1-(-3)}$

$= \dfrac{2^3-1-(-2)^3+1}{4} = \dfrac{16}{4} = 4$

$\exists$ a value c where $f'(c)=4$

$f'(x) = 3(x+1)^2 \cdot 1$
$= 3(x+1)^2$

solve: $3(x+1)^2 = 4$

$(x+1)^2 = 4/3$

$x+1 = \pm\sqrt{4/3}$

$x = \pm\sqrt{4/3} - 1$

$= \pm\frac{2}{\sqrt{3}} - 1 = c$

both of these pts are in the interval.

We found 2!

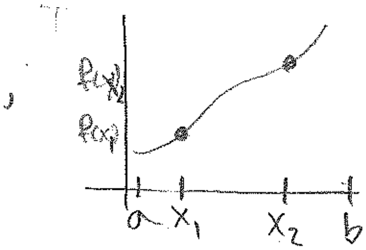
Proof of MVT comes from Rolle's Thm. That's what we'll do now

MVT & increasing/decreasing

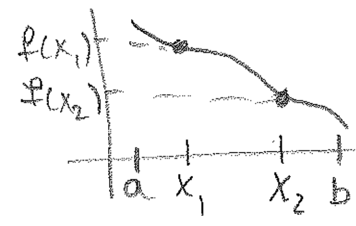
I = interval ((a,b), $[a,b)$, $(a,b]$, $[a,b]$)

DEF:

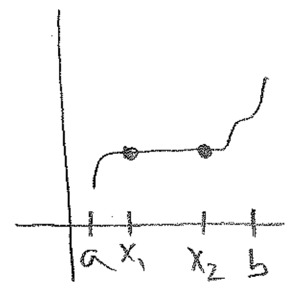
(1) $f(x)$ is increasing on I if for any numbers, that are in I $f(x_1) < f(x_2)$ whenever $x_1 < x_2$



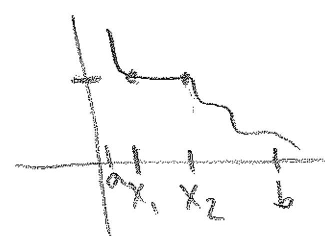
(2) $f(x)$ is decreasing on I if " " $f(x_1) > f(x_2)$ whenever $x_1 < x_2$



(3) $f(x)$ is nondecreasing on I if $f(x_1) \leq f(x_2)$ whenever $x_1 < x_2$



(4) $f(x)$ is nonincreasing on I if $f(x_1) \geq f(x_2)$ whenever $x_1 < x_2$



Recall: decreasing func $\Rightarrow$ neg der.
increasing func $\Rightarrow$ pos der.

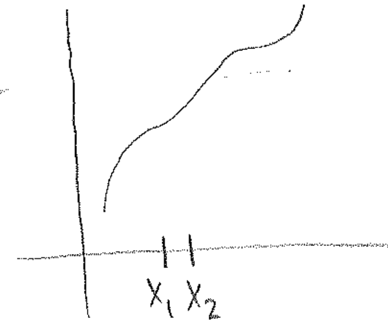
In theorem below: I is an interval
$J = I$ minus endpts.
If $I = [a,b], [a,b), (a,b], (a,b)$
then $J = (a,b)$

<u>Thm</u>: (1) If $f'(x) > 0$ for all x in J, f is increasing on I
(2) If $f'(x) < 0$ for all x in J, f is decreasing on I
(3) If $f'(x) \geq 0$ " f is nondecreasing on I
(4) If $f'(x) \leq 0$ " f is nonincreasing on I.

Seems obvious... but how do we really know?

MVT!

take two pts very close together



we know $\exists$ c with

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = f'(c)$$

$$\underbrace{f(x_2)-f(x_1)}_{\substack{\text{pos if inc} \\ \text{neg if reg}}} = (x_2-x_1)\underbrace{f'(c)}_{\substack{\text{pos if inc} \\ \text{neg if dec}}}$$

Determining where a Function Inc/Dec

Ex: On what intervals is $f(x)=x^3+x+1$ increasing or decreasing?

Step 1: Calculate derivative

$f'(x) = 3x^2+1$

Step 2: Decide where der is pos, neg, zero

* set $f'(x)=0$ and solve for x

$3x^2+1=0$

$3x^2=-1$

$x^2=-\frac{1}{3}$ no solutions so never equals zero

always positive: $(-\infty,\infty)$

Step 3: f is increasing from $(-\infty,\infty)$

Ex: When is $f(x)=2x^3-6x^2-18x+1$ inc/dec?

Step 1: derivative $-$ $f'(x)=6x^2-12x-18$

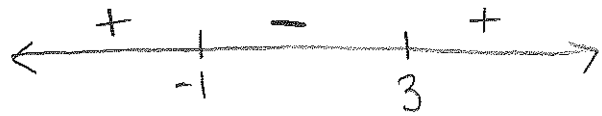
Step 2: where is der. pos/neg/zero?

* solve $f'(x)=0$

$$6x^2-12x-18=0$$

$$(6x+6)(x-3)=0 \quad x=-1 \text{ OR } 3$$

Number line:
Test for pos/neg in each region



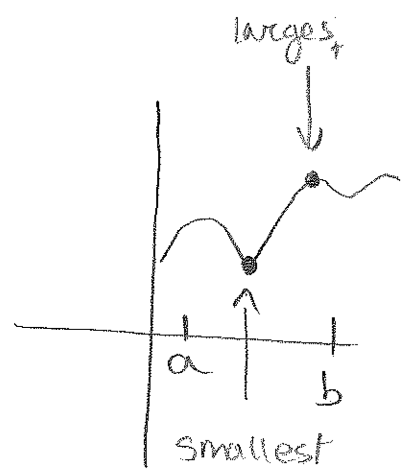
Step 3: Bring back to $f(x)$

f is inc. on $(-\infty,-1] \cup [3,\infty)$

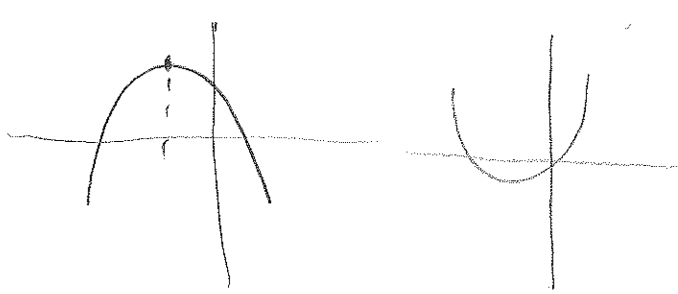
dec on $[-1,3]$

Some Theorems

1. f cont on $[a,b]$ then there exists a point in the interval where f is largest (maximized) and a pt where f is smallest (minimized)



2. The max/min values will occur at one of the following
   (a) an endpt ($x=a$ OR $x=b$)
   (b) a critical pt, c, where $f'(c)=0$



Rolle's Thm: (Proves MVT)

- f cont on $[a,b]$
- f diff'ble on (a,b)
- $f(a)=f(b)=0$ (a,b are roots of f)

Then there exists a value c in (a,b)
where $f'(c)=0$

i.e. Since f is "nice" it had to turn around somewhere!